A Feasible-region based Homotopy-enhanced Interior Point Optimal Power Flow

October 5, 2021

Prof. Hsiao-Dong Chiang

GOT-BSI-OPF Team

1.School of ECE, Cornell University, NY 14850, USA,

2. Bigwood Systems Inc. and Global Optimal Technology, Inc. Ithaca, NY 14850

General Optimal Power flow Formulation

$$
\begin{aligned}
&\min && f(V,\theta,t,\phi,b,P^G,Q^G) \\
&s.t. && P_i(V,\theta,t,\phi,b) + P_i^L - P_i^G = 0, i = 1,\cdots,n_B \\
& && Q_i(V,\theta,t,\phi,b) + Q_i^L - Q_i^G = 0, i = 1,\cdots,n_B \\
& && S_{ij}(V,\theta,t,\phi,b) \le S_{ij}, (i,j) \in L \\
& && S_{ji}(V,\theta,t,\phi,b) \le \bar{S}_{ij}, (i,j) \in L \\
& && \underline{V_i} \le V_i \le \overline{V_i}, i = 1,\cdots,n_B \\
& && \underline{t_i} \le t_i \le \overline{t_i}, i = 1,\cdots,n_T \\
& && \underline{\phi_i} \le \phi_i \le \overline{\phi_i}, i = 1,\cdots,n_P \\
& && \underline{b_i} \le b_i \le \overline{b_i}, i = 1,\cdots,n_S \\
& && \underline{P_j^G} \le P_j^G \le \overline{P_j^G}, j = 1,\cdots,n_G \\
& && \underline{Q_j^G} \le Q_j^G \le \overline{Q_j^G}, j = 1,\cdots,n_G
\end{aligned}
$$

Equality constraint s

$C_E(x) = 0$

Inequality constraint s

$C_I(x) \le 0$

General Optimal Power flow Formulation

$$
\begin{aligned}
\min \quad & f(V,\theta,t,\phi,b,P^G,Q^G) \\
s.t. \quad & P_i(V,\theta,t,\phi,b) + P_i^L - P_i^G = 0, i = 1,\cdots,n_B \\
& Q_i(V,\theta,t,\phi,b) + Q_i^L - Q_i^G = 0, i = 1,\cdots,n_B \\
& S_{ij}(V,\theta,t,\phi,b) \le S_{ij}, (i,j) \in L \\
& S_{ji}(V,\theta,t,\phi,b) \le \bar{S}_{ij}, (i,j) \in L \\
& \underline{V}_i \le V_i \le \bar{V}_i, i = 1,\cdots,n_B \\
& \underline{t}_i \le t_i \le \bar{t}_i, i = 1,\cdots,n_T \\
& \underline{\phi}_i \le \phi_i \le \bar{\phi}_i, i = 1,\cdots,n_P \\
& \underline{b}_i \le b_i \le \bar{b}_i, i = 1,\cdots,n_S \\
& \underline{P}_j^G \le P_j^G \le \bar{P}_j^G, j = 1,\cdots,n_G \\
& \underline{Q}_j^G \le Q_j^G \le \bar{Q}_j^G, j = 1,\cdots,n_G
\end{aligned}
$$

Equality constraints

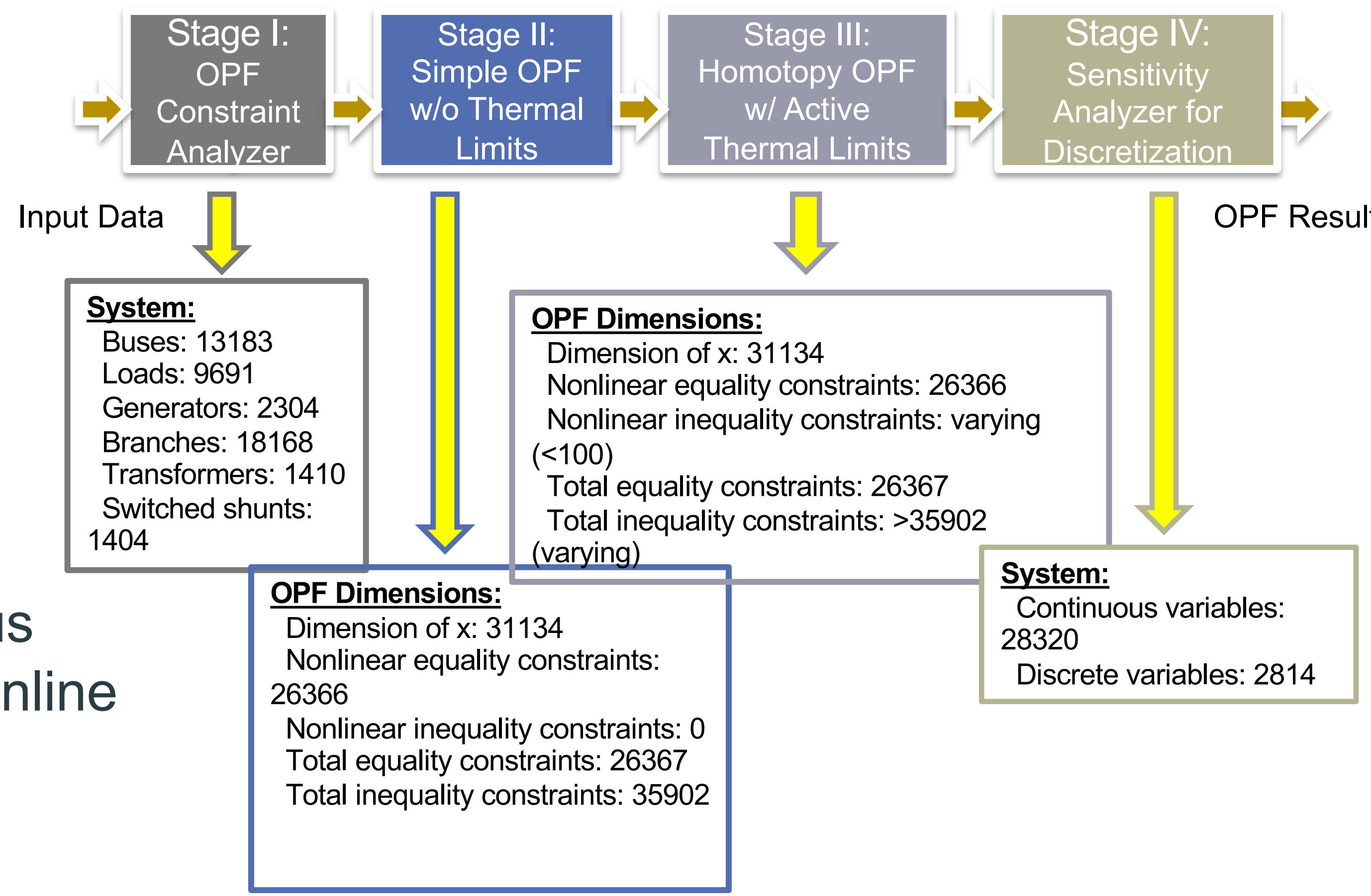
$C_E(x) = 0$

Inequality constraints

$C_I(x) \le 0$

3

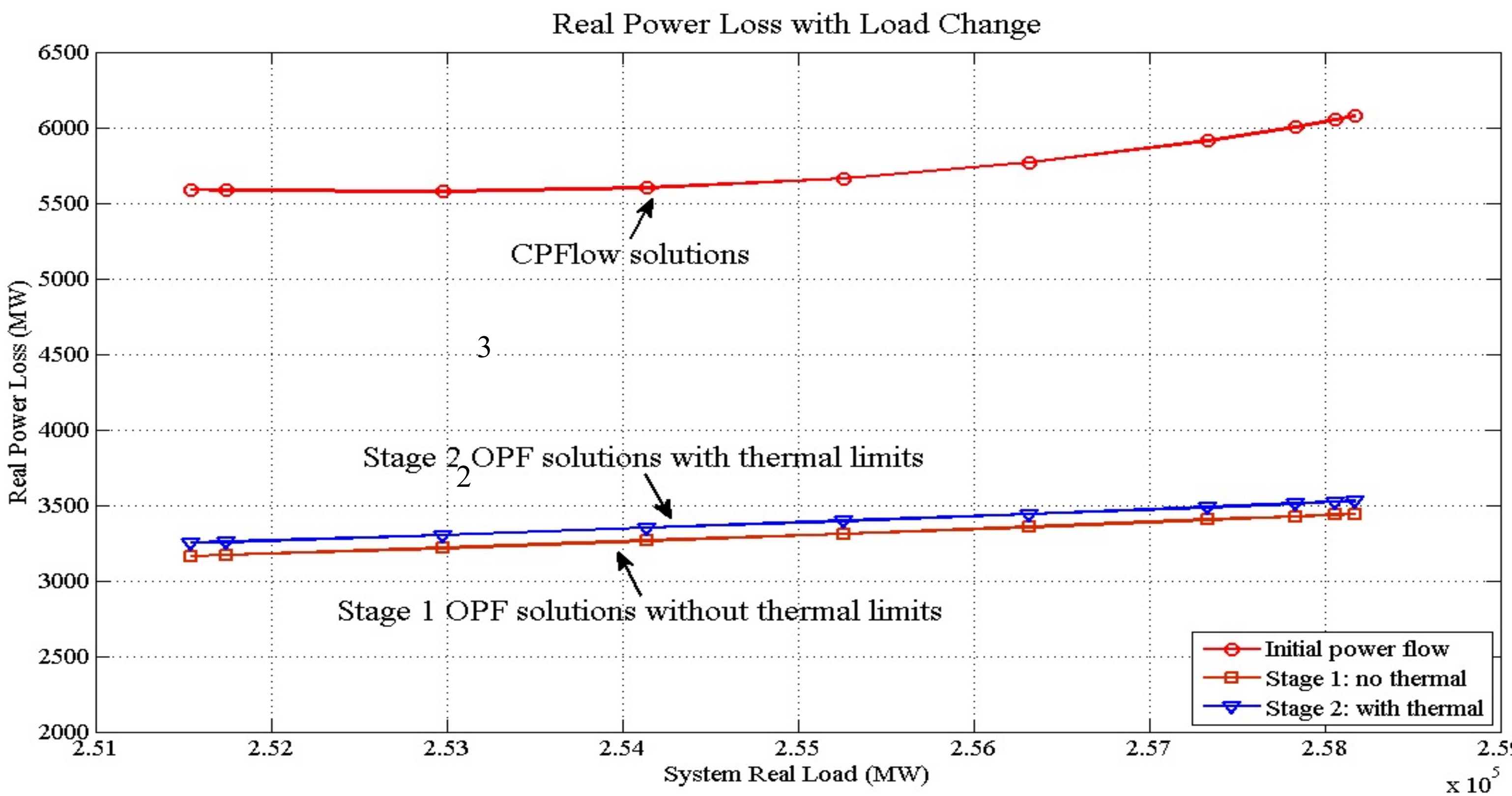
A Novel Homotopy-enhanced IPM to deal continuous and discrete decision variables.



13183-Bus System (online case)

Results: Real Power Loss Reductions (45% reduction) for 10 operating conditions

- Online data : from SCADA to State Estimator (real-time data) and apply a Continuation Power Flow to generate addition heavy loading operating points



5

ROBUSTNESS

Two-Stage OPF method vs. Single-Stage Interior Point Method

Robustness of our method

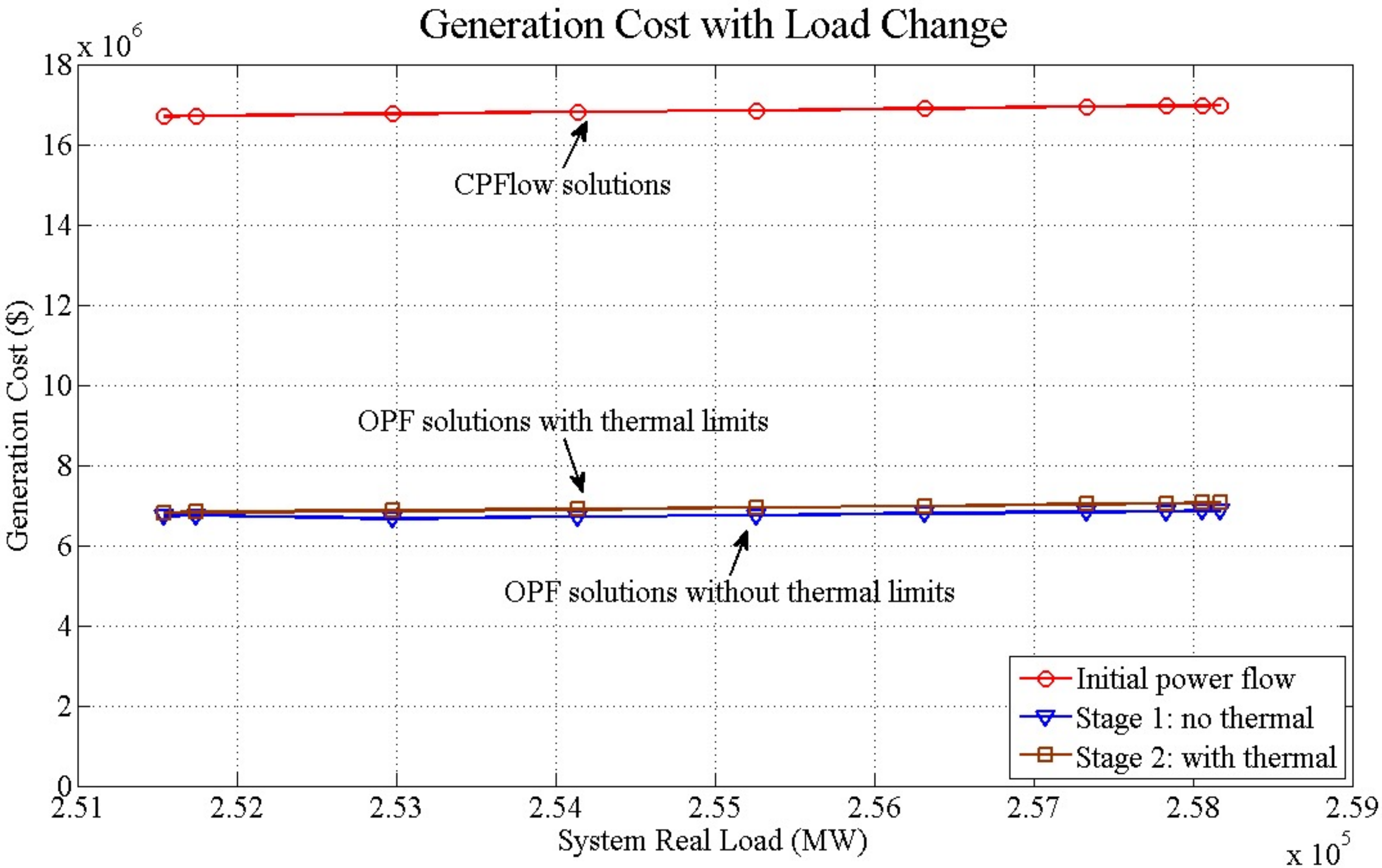
Loading Condition	One-Staged Scheme	Multi-Staged Scheme
1	Succeeded	Succeeded
2	Succeeded	Succeeded
3	Succeeded	Succeeded
4	Succeeded	Succeeded
5	Failed	Succeeded
6	Failed	Succeeded
7	Failed	Succeeded
8	Failed	Succeeded
9	Failed	Succeeded
10	Failed	Succeeded

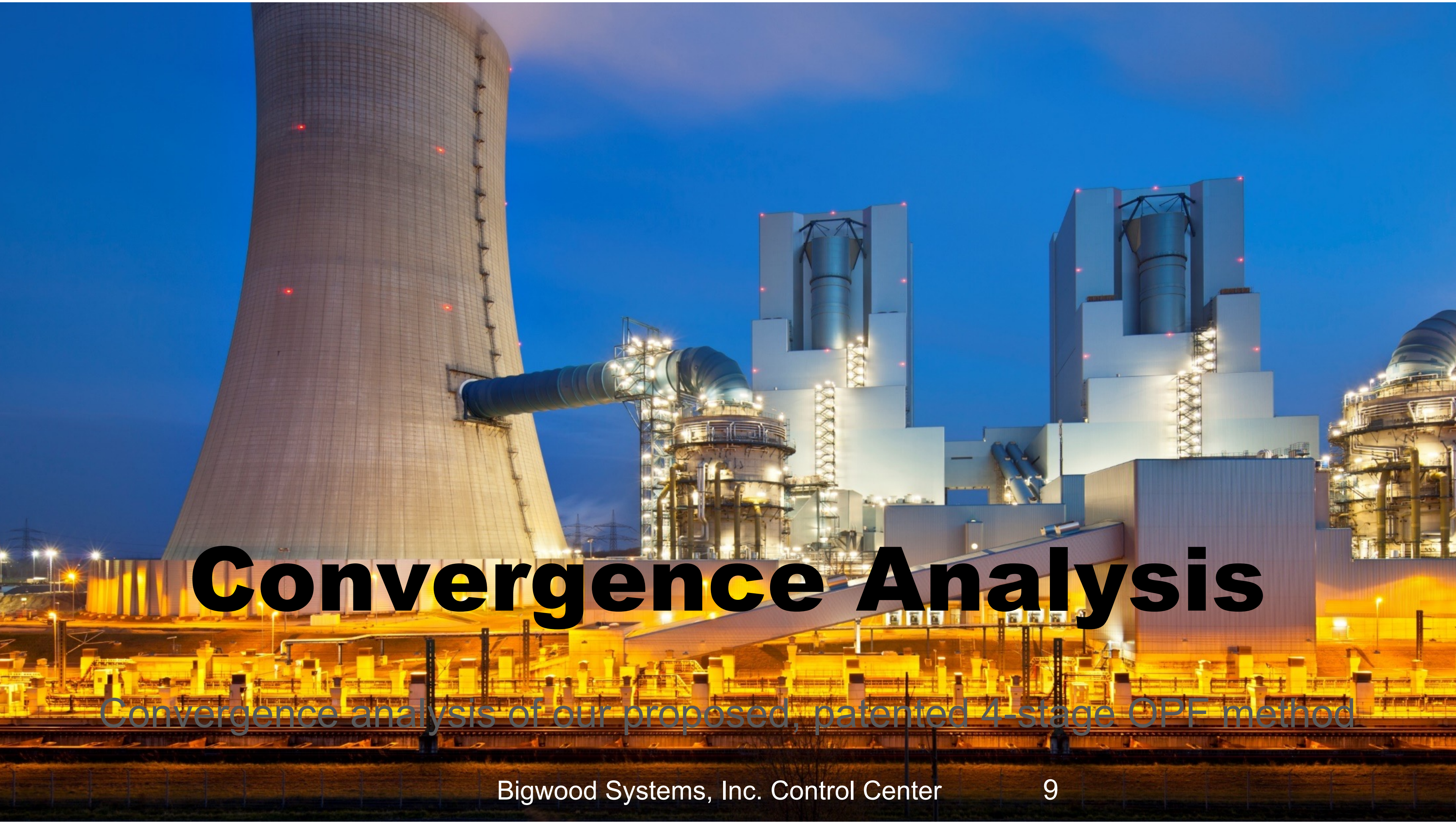
Result 2: Generation Cost Minimization (40% reduction is possible)

- Randomly assigned generator types and costs

Type	# of Generators	Min Cost ($/MWHr)	Max Cost ($/MWHr)	Mean Cost ($/MWHr)
Coal	40 %	11.0	282.0	26.0
Oil	15 %	31.0	1040.0	284.0
Nat. Gas	15 %	53.0	317.0	96.0
Hydro	26 %	0.0	0.0	0.0
Nuclear	2 %	0.0	0.0	0.0
Wind	2 %	0.0	0.0	0.0

Result 2: Generation Costs





Convergence Analysis

Convergence analysis of our proposed, patented 4-stage OPF method

On the Global Convergence of a Class of Homotopy Methods for Nonlinear Circuits and Systems

Tao Wang, *Member, IEEE*, and Hsiao-Dong Chiang, *Fellow, IEEE*

***Abstract*—Homotopy methods are developed for robustly computing solutions of nonlinear equations, which is of fundamental importance in nonlinear circuit and system simulations. This brief develops theoretical results on the global convergence of a class of homotopy methods for solving nonlinear circuits and systems. A set of sufficient conditions that guarantee the global convergence of homotopy methods is derived. These analytical results are then illustrated on a small nonlinear circuit and a large (about 10 000-dimension) power grid.**

***Index Terms*—Convergence theorem, homotopy-based method, power flow equations, power grid.**

I. Introduction

HOMOTOPY methods (also called trend embedding or

power flow studies of practical power grids that have more than 10 000 nodes [17].

The convergence property plays an important role in evaluating solution methods. For instance, a global convergence theorem has been developed [18] for a family of path-following methods, which can determine all the solutions to a system of nonlinear equations satisfying a given sufficient condition. However, this condition cannot be directly checked for nonlinear circuits and systems. Moreover, globally convergent probability-one homotopy methods have been applied to the dc operating problems, and certain "coercivity" conditions are checked when investigating the convergence [11], whereas a tailored homotopy method is devised for the same problem to avoid the shortcomings caused by turning-point nesting [9].

US009964980B2

(12) **United States Patent**
Chiang et al.

(10) **Patent No.: US 9,964,980 B2**
(45) **Date of Patent: May 8, 2018**

(54) **METHOD AND APPARATUS FOR OPTIMAL POWER FLOW WITH VOLTAGE STABILITY FOR LARGE-SCALE ELECTRIC POWER SYSTEMS**

(71) Applicant: **Bigwood Technology, Inc.**, Ithaca, NY (US)

(72) Inventors: **Hsiao-Dong Chiang**, Ithaca, NY (US); **Bin Wang**, Ithaca, NY (US)

(73) Assignee: **Bigwood Technology, Inc.**, Ithaca, NY (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 704 days.

2012/0179301 A1*	7/2012	Aivaliotis	....	H02J 3/00 700/286
2013/0018517 A1*	1/2013	Kalagnanam	....	G06Q 50/06 700/291
2014/0046493 A1*	2/2014	Sun	....	G05B 15/02 700/286
2015/0051744 A1*	2/2015	Mitra	....	G06Q 50/06 700/286

(Continued)

OTHER PUBLICATIONS

Wu et al., "A direct nonlinear predictor-corrector primal-dual interior point algorithm for optimal power flows", IEEE Transactions on Power Systems, vol. 9, No. 2, pp. 876-883, May 1994.

(Continued)

General Optimal Power flow Formulation

$$
\begin{aligned}
\min \quad & f(V,\theta,t,\phi,b,P^G,Q^G) \\
s.t. \quad & P_i(V,\theta,t,\phi,b) + P_i^L - P_i^G = 0, i = 1,\cdots,n_B \\
& Q_i(V,\theta,t,\phi,b) + Q_i^L - Q_i^G = 0, i = 1,\cdots,n_B \\
& S_{ij}(V,\theta,t,\phi,b) \le S_{ij}, (i,j) \in L \\
& S_{ji}(V,\theta,t,\phi,b) \le \bar{S}_{ij}, (i,j) \in L \\
& \underline{V}_i \le V_i \le \bar{V}_i, i = 1,\cdots,n_B \\
& \underline{t}_i \le t_i \le \bar{t}_i, i = 1,\cdots,n_T \\
& \underline{\phi}_i \le \phi_i \le \bar{\phi}_i, i = 1,\cdots,n_P \\
& \underline{b}_i \le b_i \le \bar{b}_i, i = 1,\cdots,n_S \\
& \underline{P}_j^G \le P_j^G \le \bar{P}_j^G, j = 1,\cdots,n_G \\
& \underline{Q}_j^G \le Q_j^G \le \bar{Q}_j^G, j = 1,\cdots,n_G
\end{aligned}
$$

Equality constraints

$C_E(x) = 0$

Inequality constraints

$C_I(x) \le 0$

12

The complete constraint functions of the OPF problem can be presented as the following equalities:

$$H(x) = 0, \quad x \in \Re^n$$

where $H = (h_1, \ldots, h_m)^T : \Re^n \to \Re^m$

Definition 1: (Feasible region)

The feasible region of general OPF problems is the set of control variables in which all the equality and inequality constraints of the problem are satisfied, i.e.,

$$FR = \{u \in \Re^{2N_G - 1} : H(x) = H(u, y(u), s(u, y)) = 0\}$$

$\dot{x} = Q_H(x) = -DH(x)^T H(x)$

Purpose: to completely characterize the feasible region of the set of nonlinear constraint functions of OPF problems.

Approach: we consider the following nonlinear dynamical system, which is closely related to the nonlinear constraint functions:

$$\dot{x} = Q_H(x) = -DH(x)^T H(x)$$

where $DH(x)$ is the Jacobian matrix of $H(x)$.
We term the above system quotient gradient system (QGS).

Theorem 4: (Complete characterization, Chiang and Jiang, 2018)

A path-connected set is a feasible component of constraint set <u>if and only if</u> it is a regular stable equilibrium manifold of QGS; i.e.,

$$FR = \bigcup_{j=1} \Sigma_j^r$$

where Σ_j^r is a regular stable equilibrium manifold of QGS.

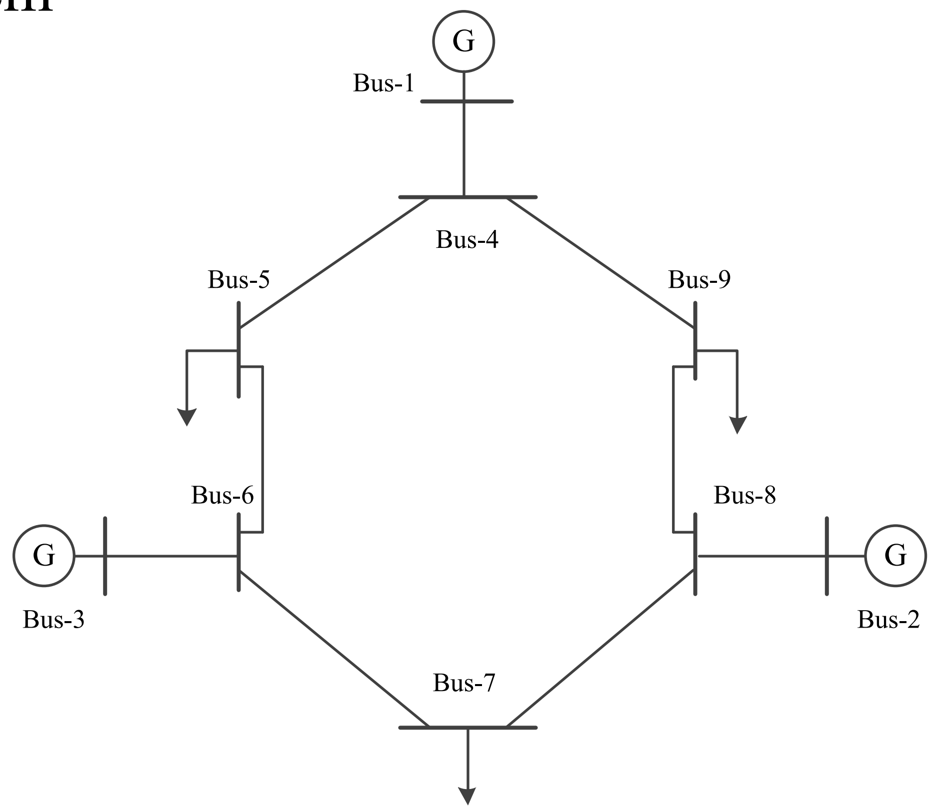
Complete stability and Global Convergence

Theorem 5: (Chiang and Jiang, 2018)

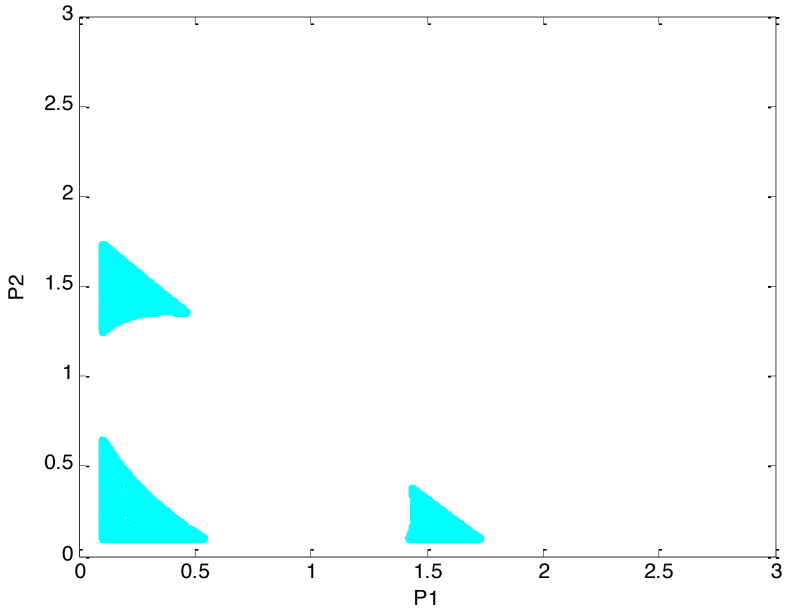
Every trajectory of QGS converges to one of its stable equilibrium manifolds.

Feasible Regions & SEMs

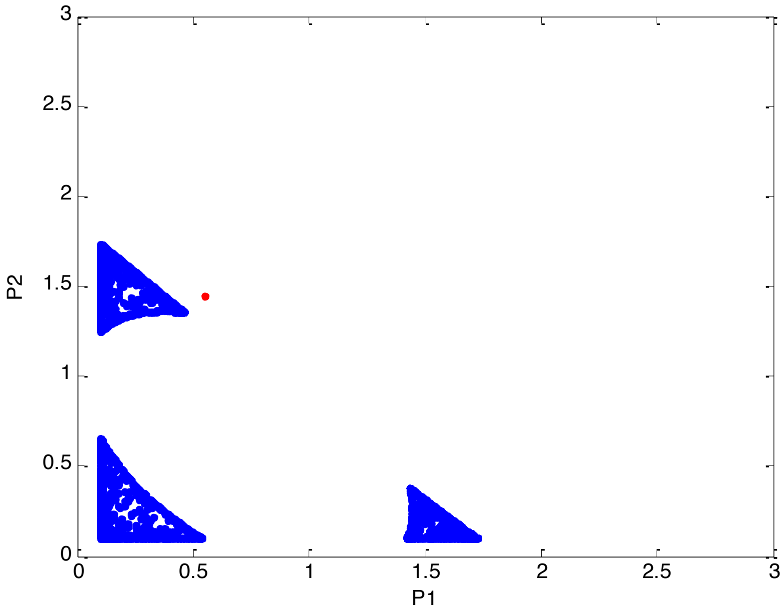
9-bus system



Feasible Regions & SEMs of 9-bus system



(a) Feasible regions on *P1-P2* projection plane

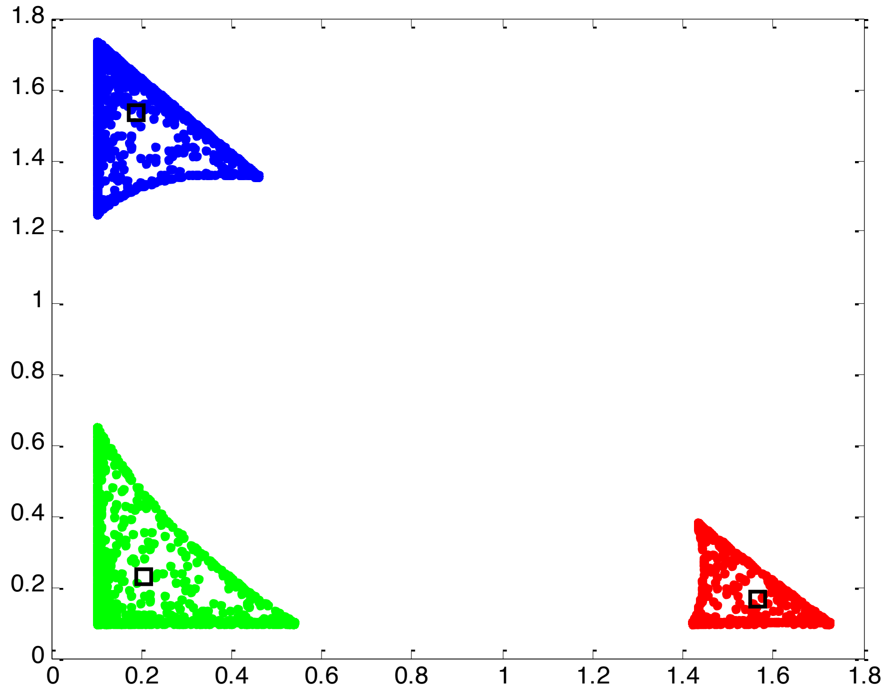


(b) SEMs on *P1-P2* projection plane

Convergence regions for feasible regions

For the 9-bus system

- There are 3 feasible regions



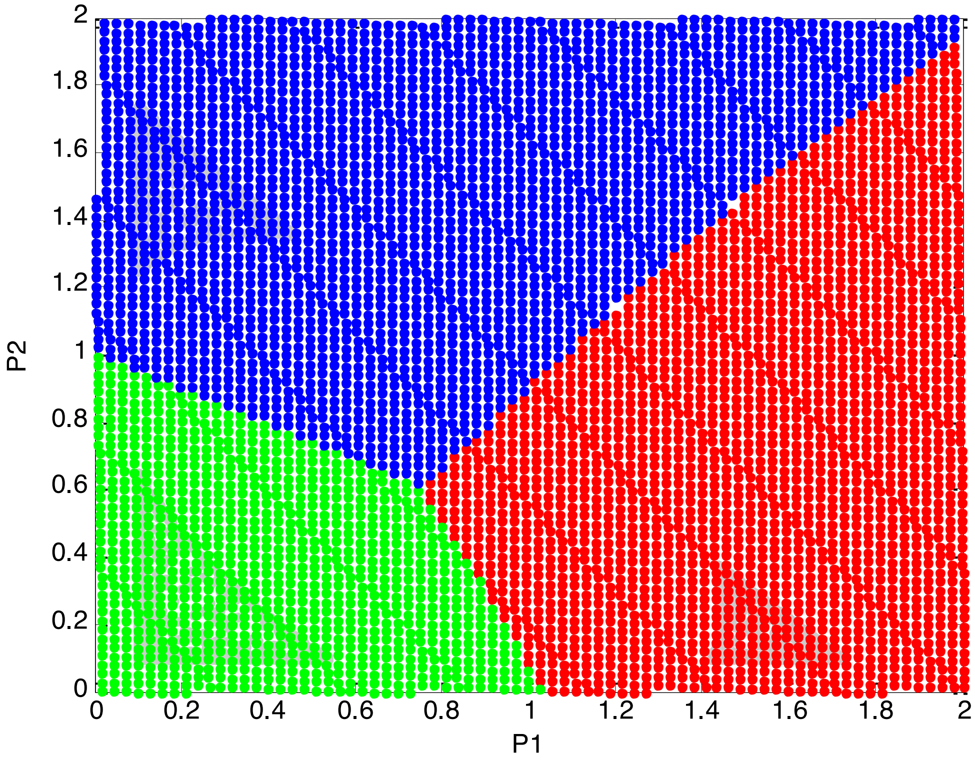
Complete stability and Global Convergence

Theorem 5: (Chiang and Jiang, 2018)

Every trajectory of QGS converges to one of its stable equilibrium manifolds.

Convergence regions for feasible regions

Convergence regions of QGS on tangent plane



Necessay and sufficient Condition for the existence of an OPF solution

Theorem : (Chiang and Wang, 2018)

Consider an OPF problem with an objective function that is continuous and subject to a set of equality and inequality constraint functions (1)-(4). Suppose the equilibrium manifolds of the QGS (11) are isolated, pseudo-hyperbolic, and finite in number. Then, an OPF solution exists if and only if there is a regular SEM of QGS (12).

Lower Bound for the Number of OPF Solutions

Theorem : (Chiang and Wang, 2018)

Consider an OPF problem with an objective function that is continuous and subject to a set of equality and inequality constraint functions (1)-(4). Suppose the equilibrium manifolds of the QGS (11) are isolated, pseudo-hyperbolic, and finite in number. Then, the number of local OPF solutions to the OPF problem is not less than the number of regular SEMs of the QGS (12).

On the Existence of and Lower Bounds for the Number of Optimal Power Flow Solutions

Hsiao-Dong Chiang, *Fellow, IEEE*, and Tao Wang, *Senior Member, IEEE*

***Abstract*—An optimal power flow (OPF) problem can have one or multiple locally optimal solutions. It can also admit no solution. Few studies have addressed the existence of OPF solutions and derived a (tight) lower bound for the number of OPF solutions. The present paper is devoted to the analytical aspects of OPF solutions. Specifically, a necessary and sufficient condition for the existence of an OPF solution is presented and a tight lower bound for the number of OPF solutions is derived; in other words, an existence theorem is developed for the OPF solution and a computable lower bound for the number of solutions is derived by investigating a dynamical system called the quotient gradient system (QGS), which is built from the set of constraints. It is shown that an OPF solution exists if and only if the QGS has a regular stable equilibrium manifold (SEM) and the number of OPF solutions is not less than the number of regular SEMs for the QGS. Two test systems with 9 and 118 buses, respectively, are evaluated, and the numerical results are summarized to validate the presented analytical results and**

lutions are limited [5]–[7]. On the other hand, numerical studies have been devoted to the existence of local OPF solutions in standard OPF test cases, and observations have been provided to explain the reasons for their existence, such as circulating flows, high line angles, and excessive reactive power [8].

An algorithm has been proposed to compute the feasible region of small OPF problems and was simulated in 5-bus and 9-bus test cases [9]. A mathematical characterization has been developed for the feasible region of the OPF problem, and an equivalence has been established between the feasible region of the OPF problem and the union of all the regular stable equilibrium manifolds (SEMs) of the quotient gradient system (QGS) [10]. The QGS has proved to be completely stable, so its trajectory always converges to an equilibrium manifold [10].